Revista EIA, ISSN 1794-1237 / Year XIII / Volume 13 / Issue N.25 / January-June / pp. 13-27
Technical-scientifical biannual publication / Universidad EIA, Envigado (Colombia)

GENERALIZED DIMENSIONAL ANALYSIS

Gabriel Poveda Ramos[1]

ABSTRACT

This paper begins by defining such concepts as measurement, measure, magnitude, and dimension, giving examples to illustrate them. Magnitudes are also addressed and defined in such a way that they may be identified in the areas of the social, natural and human sciences, in addition to those magnitudes usually accepted in the physical sciences. Some mistaken concepts are corrected relating to the dimensions of physical quantities such as force, plane angle, magnetism and entropy. The paper also presents other concepts which are often ignored in physics textbooks, and the many magnitudes which are plainly ignored in teaching social and natural sciences.

The nature of vector space has the kind of magnitudes that appear in all the sciences mentionedwith regard to the operation of internal composition between magnitudes and external composition with the class of rational numbers. With an example taken from the theory of project evaluation, it shows the great utility these concepts bring to the discipline of dimensional analysis, as with Lord Kelvin's algorithm for the deduction of quantitative laws for physical, social, economic and other phenomena that can be analyzed with the pi theorem of Buckingham, Vaschy and Ostrogradsky.

PALABRAS CLAVE: Dimensional analysis; Measure; Measurement; Magnitude; Dimension; Physical Sciences; Social Sciences; Natural Sciences; Pi Theorem.

ANÁLISIS DIMENSIONAL GENERALIZADO

RESUMEN

El artículo comienza por definir los conceptos de medición, medida, magnitud, dimensión, ilustrándolos con ejemplos. Además se mencionan magnitudes así definidas que se pueden identificar en el mundo de las Ciencias Sociales, las Ciencias Naturales, las Ciencias Humanas, además de las magnitudes que usualmente se aceptan en las Ciencias Físicas. Se corrigen conceptos equivocados sobre las dimensiones de magnitudes físicas como Fuerza, Ángulo plano, Magnetismo y Entropía, y se presentan otros conceptos que suelen ser ignorados en los libros de Física y las muchas magnitudes que son simplemente ignoradas en Ciencias Sociales y en Ciencias Naturales.

Se pone de presente la naturaleza de Espacio Vectorial que tiene la clase de las magnitudes que aparecen en todas estas ciencias frente a la operación de composición interna entre magnitudes, y la de composición externa con la clase de los números racionales, y con un ejemplo tomado de la teoría de la Evaluación de Proyectos, se muestra la gran utilidad

[1] Chemical Engineer, Electrical Engineer, Mathematician. Doctor of Engineering.

Author of correspondence: Poveda Ramos, G. (Gabriel). Email: gapora@une.net.co

Paper history:
Paper received: 01-XI-2015 / Approved: 11-VII-2016
Available online: October 30 2016
Open discussion until October 2017

DOI: http:/dx.doi.org/10.14508/reia.2016.13.25.13-27

que aportan estos conceptos a la disciplina del Análisis Dimensional, como ocurre con el algoritmo de Lord Kelvin para la deducción de leyes cuantitativas para los fenómenos físicos,sociales, económicos y otros que son susceptibles de analizar con el Teorema Pi de Buckingham-Varschy y Ostrogradsky.

PALABRAS CLAVE: análisis dimensional; medida; magnitud; medición; dimensión; Ciencias Físicas; Ciencias Sociales; Ciencias Naturales; Teorema Pi.

ANALISE DIMENSIONAL GENERALIZADO

RESUMO

O artigo começa por definir os conceitos de medição, medida, magnitude, dimensão, ilustrando-os com exemplos. Também se mencionam magnitudes assim definidas que podem ser identificadas no mundo das Ciências Sociais, Ciências Naturais, as Ciências Humanas, além das magnitudes que normalmente se aceitam nas CiênciasFísicas. Se corrigem conceitos equívocos sobre as dimensões de magnitudes físicas como Força, ângulo plano, Magnetismo e entropia, e se apresentam outros conceitos que geralmente são ignorados nos livros de física e as muitas magnitudes que são simplesmente ignoradas em Ciências Sociais e Ciências Naturais.

Ergue-se de presente a natureza de espaço vectorial que tem o tipo das magnitudes que aparecem em todas essas ciências frente à operação de composição interna entre magnitudes e a composição externa com a classe dos números racionais, e com um exemplo da teoria da avaliação do projetos, se apresenta a grande utilidade mostrado que aportam conceitos à disciplina de Análise Dimensional, como ocorre com o algoritmo de Lord Kelvin para a derivação de leis quantitativas para os fenômenos físicos, sociais, econômicos e outros que são susceptível de analisar com o Teorema Pide Buckingham-Varschy e Ostrogradsky.

PALAVRAS-CHAVE: Análise dimensional; A medida; Medição; Magnitude; Dimensão; Ciências físicas; Ciências Sociais; Ciências Naturais; Pi Teorema.

1. INTRODUCTION

This is a fundamental chapter that one day will be called metascience or science of the sciences. Its purpose is to show the unification of the quantifiable knowledge of humanity (some of them being very developed, while others are in their infancy), whichcan be done through analysis and dimensional analysis algorithms.

This is a fundamental chapter in the theory of knowledge that until today would remain unknown and for which this document tries to build a rigorous terminology with words that are common in ordinary language such as "magnitude," "dimensions," and "product," it being indispensable to define and use such words in this context.

To rigorously construct the dimensional analysis in which this new chapter of abstract algebra on vector spaces is supported,which makes it possible to establish numerous properties within the discipline of the theory of knowledge as the author has demonstrated it of this note over the course of several years of work.

It should be remembered that today some sciences (such as hydrodynamics, optics, meteorology, etc.) lend themselves more than others (such as edaphology, the theory of color and others) to the formulation of problems and quantitative theorems through dimensional analysis. But it is foreseeable that, with the general advance of human knowledge, these disparities will balance out as new measuring instruments, new knowledge of reality and new theories on the world around us become available.

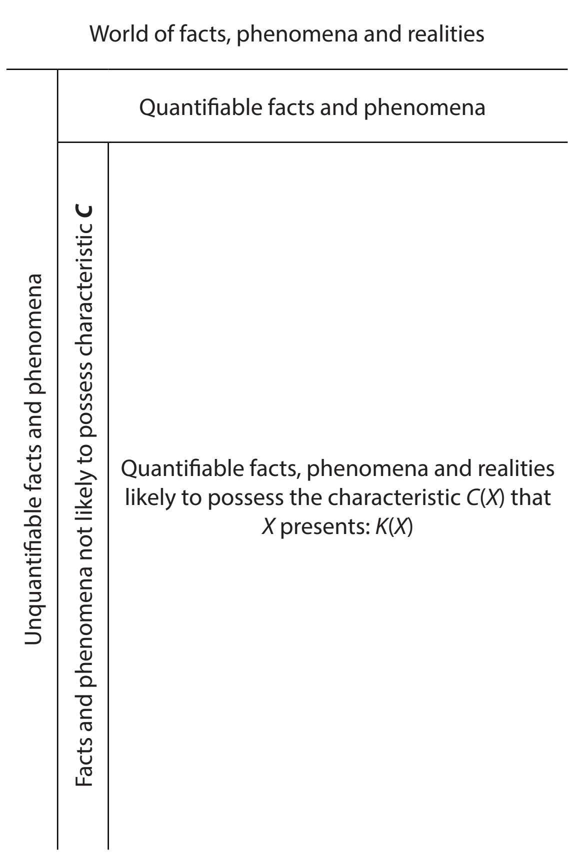
2. GENERALIZED DIMENSIONAL ANALYSIS

Measurable properties and characteristics

Human beings—as a people and as a society—find, when addressing nature and their own societies, a multitude of objects, facts and phenomena that repeatedly occur in their experience and that present to them some property or characteristic which is analogous, qualitatively, to one case or another. We refer to such objects and facts as "entities," and they can be of a physical, natural, social or human nature.

Each fact, object or phenomenon exhibits characteristics—one or many—that can be qualitative (such as the beauty of a sculpture) or subjected, each one, to a procedure of grading or scaling (like solid body weight, the conductivity of metals, the height of a person, the intensity of electric currents, the population of a country, etc.). The latter are examples of quantifiable characteristics, which will be designated with the symbol *C*, An object or phenomenon that is likely to possess said characteristicwill be designated by *X*, so that the symbol *C(X)* signifies "quantifiable characteristic *C* that corresponds to the fact or object *X*". And the class of the objects having the same characteristic *C*(*X*)which *X*, has, be it to a greater or lesser degree or the same degree as *X*, will be called *K*. This last is, then, the class of all the facts, phenomena or processes that present the quantifiable characteristic *C* which object *X* has.

TABLE 1. MEASURABLE PROPERTIES AND FEATURES



Given thusly a class $K(X)$ of facts X that possess or present a quantifiable property or characteristic C, it is possible to submit each pair of elements of said class (those we designate X_1 and X_2) to empirical comparisons to establish only one of three possibilities: a) either $C(X_1) > C(X_2)$; b) or $C(X_2) > C(X_1)$; c) or $C(X_1) = C(X_2)$, for all pairs of elements X_1 and X_2.

To perform the comparisons mentioned it is necessary and sufficient to stipulate:

1. The two individuals A and B that are compared regarding their characteristic in common C (their weights, lengths, numbers, areas).

2. Apparatuses and methods for empirical comparison of $C(A)$to $C(B)$, such as scales, straight lines or measuring tape, counters, technical standards,

optical instruments, etc. Among such resources it is often necessary to count with one or various prototypes of the unit in which one wants to express the characteristic to be compared between A and B.

3. In particular, a criterion or condition to determine when two individuals A_1 and A_2 have equal characteristics: $C(A_1) = C(A_2)$, or when $C(A_1)$ is $C(A_1) > C(A_2)$ or vice versa.

In the physical world there are many characteristics or properties to which these methodological operations are applicable, and they likewise belong to a wide array of classes of objects and facts. For example: the weight of pieces of matter, the volume of solid bodies, the temperature of liquids, the electric charge of electrically charged bodies, the entropy of vapor masses, the light intensity of light sources, the concentration of solutions, etc. Traditional physics texts haveerroneously limited these fundamental properties of the physical world to characteristics such as length, mass, duration, electric charge and (at most) temperature. They have mistakenly claimed that angular amplitudeshave no dimensions, that entropy is like the inverse of an absolute temperature, and other similar errors.

The characteristics or properties of the physical world that lend themselves to comparison between what corresponds to each of two different objects or phenomena that possess it (as described above) are said to be measurable. In the set of methodological operations numbered above with the digits 1, 2 and 3, it is called a measurement of the measurable property C; and the result of each measurement of C is called a (specific) measure of C. For example: 2 cm is a (specific) measure of distance; 100 ° K is a (specific) temperature measurement; 525 Btu is a measure of the amount of heat; etc.

The trichotomy mentioned above gives rise to a one-to-one correspondence between the class of elements $\{C(X)\}$ and the class R_+ of positive real numbers. Constructing a correspondence such as that mentioned is called measuring the characteristic C in the element X of $K(X)$.

This procedure consists of the following steps:

1. Choose an element u of class $K(X)$ that is clearly definable and universally acceptable as a unitary pattern for all elements of $K(N)$. This is the case for meter to measure lengths in the metric system; it is the case for the ampere-hour to measure quantities of electricity in the Giorgi system; and it is the case for the dollar to measure amounts of money in the world economy.

2. Design and protocolize an empirical procedure to compare $C(X)$ with $C(u)$ to determine the ratio p/q that is equal to $C(X)$: $C(u)$.

3. Conclude that $C(X) = (p/q) * C(u)$ where the "*" symbol means "p/q times greater" than the standard unit $C(u)$. This is the measure of $C(X)$ in units of u.

A well-defined quantifiable discipline (e.g. demography, microeconomics, geodesy, etc.) needs to have and use a system of units that, for pragmatic reasons, must be universally used. These units should be those that refer to the fundamental magnitudes of each discipline in question. Thus, in the case of microeconomics, whose fundamental magnitudes are population, money and civil time, it is necessary to define and use units such as "thousand persons," "dollar" and "year." In thermodynamics, whose fundamental magnitudes are temperature, mechanical work and amount of heat, it is necessary to accurately define and use the "kelvin," "erg" and "calorie." While in finance it is necessary to rigorously define what "dollar," "year" and "1% in arrears" are.

Any measure of any measurable property (such as mass, length, entropy, etc.) is formed by a real number (which is usually a rational or broken number, such as p/q) that multiplies operationally to a unit of measure. For example: 5.2 * calories, 22 *steradians; 150.3 kilometers; 3,250 * man-hours; etc.

Measurable magnitudes

The class of all possible measures, whether real or hypothetical, of the same characteristic, in all the facts or objects likely to possess it (i.e. all measures of duration, real and possible) is what is called a magnitude. Example: the class of all the data of the population of Colombia in all its cities and in all its epochs is, by definition, what is called "the magnitude of the Colombian urban population." The class of the distances between all the pairs of points belonging to a Euclidean space is called (simply) the Euclidean distance magnitude. And the class of all amounts of international currencies is the currency magnitude.

Natural scientists have so far been ignorant of the fact that in the world of nature there are measurable magnitudes, that they can use the algebra of magnitudes to deduce properties and quantitative relations between them, just as physicists have traditionally done in their subjects. In fact, in the case of natural sciences, there is a place for defining, among others, the following magnitudes that play an important role in such sciences: biomass, fixed carbon per hectare, BOD (biological oxygen demand), animal population, solar radiation per hectare, fixed nitrogen, precipitation per hectare per year, etc.

Traditionally, social scientists have ignored the fact that, in the world of human communities, measurable magnitudes can be identified that would allow the construction of numerous quantitative resources not presently common in these sciences. Some of these measurable magnitudes are human population, fixed capital, civil time, markets for physical goods (disaggregated by goods), agricultural land, human labor, currency, opportunity and other magnitudes that make it possible to establish quantitative relationships of an economic or social nature.

In physics and the natural and human sciences, there are magnitudes that can be called unit (or primary) magnitudes because they are very simple to define and measure, and because they can operate in the definitions of many other more complex magnitudes. This is the case forinertial mass, electric charge, temperature and entropy in physics. In the social sciences, it applies tohuman population, currency and civil time, and it is the case for biomass, rainfall and BOD in the natural sciences. **Table 1** lists the magnitudes that the author (G.P.R.) has become cognizant of through his studies of these disciplines, including unit magnitudes in the physical, natural, human and social sciences.

The whole of the unit magnitudes involved in a problem (or phenomenon or discipline) is collectively called "the dimensions of the problem (or phenomenon, or discipline)." For example, in the study of the lengths of the segments of straight lines in a plane, and of the figures that form said segments, two unit magnitudes are involved: 1) the straight line segments in the plane; 2) the plane angles between these lines. This will be called a discipline of two magnitudes. But if the forces acting on that plane also come into play, this new discipline will be said to encompass three unit magnitudes, two of which are the segments of lines and the angles (planimetry), and one of which is that of forces (dynamometry).

Unit magnitudes

Traditional physics textbooks have recognized length, mass, duration, electric charge and (some) temperatures (with qualifications) as unit or fundamental magnitudes. None have recognized natural numbers, the flat angle, the solid angle, the quantity of heat, nor many other magnitudes that are fundamental for the study and knowledge of the facts, phenomena and quantifiable realities of the world we perceive.

What is more, biology texts are still working throughrecognition ofdemography (human population), luminosity (light sciences), population (fundamental tosocial science), labor (human labor, essential in social science and economics) and many others as fundamental magnitudes.

The author of this article has identified 23 identifiable disciplines as unit sciences based on quantifiable fundamental quantities, which are defined in the table below.

TABLE 2. FUNDAMENTAL MAGNITUDES IN THE PHYSICAL, NATURAL, SOCIAL AND HUMAN SCIENCES

FUNDAMENTAL MAGNITUDE	SYMBOL	NAME	HUMAN PERCEPTION	MEASUREMENT EQUIPMENT AND METHODS	USUAL UNITS OF MEASUREMENT	OWN SCIENCE	OBSERVATIONS
Cardinality	N	Nuja	Counting objects	Counters; enumeration	Units, tens, thousands, millions, etc.	Descriptive statistics	Identified by the author (GPR)
Length	L	Distance	Topometry; micrometry	Graduated rule canvases; nanoscale	Meters, kilometers, micros, etc.	Longimetry	Also: longimétrica (longimetrics) in Spanish
Ponderal mass	M	Ponderality	Object weight	Weighing scale; scale	Kilograms, tons, pounds, grams, others	Newtonics	Do not confuse this with gravitational mass
Force and weight	F	Tare	Dynamometrics	Dynamometer; piezometer	Dynas, poundals, etc.	Dynamometry	Strength is not always proportional to mass
Flat angle	A	Angular amplitude	Goniometry	Conveyer; goniometer	Grades, radians, thousandths	Goniometry	Arc / radius definition is not appropriate
Solid angle	Ω	Solid opening	Projectometry	Stereo-goniometer	Entrerradian, Grade Square	Stereogonometry	The definition of area / square radius is not appropriate
Chronometric duration	T	Newtonian time	Chronometry	Watches; chronometers	Time and its divisions	Chronometry	It's about time on watches
Temperature	θ	Thermometry	Thermo-aesthesia and thermometrics	Thermometer; pyrometers	kelvins; degrees Rankine	Thermometry	Absolute temperature
Amount of heat	Q	Thermodynamics	Heating and cooling	Calorimeters	Small calorie, Btu, large calorie	Thermology	The amount of heat can not be assimilated with work
Entropy	S	Disorganization	Perception and counting	Multicontrollers	Hartley	Entropyology	It can be called disorder
Electric charge	E	Electric load	Electrical phenomena	Electrometers	Coulombs, electron	Electrognosia	
Magneticity	H	Immanation	Magnetic phenomena	Magnetometer	Gilbert	Magnetism	There may be magnetisms without electric fields
Brightness	Φ	Luminance	Light phenomena	Photometer	Spark plug	Photomatics	Fundamental magnitude little or not recognized
Chemical quantity	χ	Chemichionics	Electrochemical cells	Electrochemical cell	Faraday	Stoichiometry	Distinct from mass

Human population	Δ	Demography	Population counts	Census and records	Kilo-people, mega-people	Demodynamics	Fundamental magnitude in social sciences
Money	$	Cash	Currency mana-gement	Manual count	Dollars, euros, etc.	Moneterics	Fundamental magnitude in economics
Social time	Σ	Chronology	Time log	Calendars	Day, month, year	Sociochronology	It is measured by calendars
Economic capital	K	Finance	Financial mana-gement	Financial accou-nting	Mega-dollars	Chrematistics	Fundamental magnitude in economics
Productive land	G	Agrometry	Agricultural land management	Tachymeter	Hectares, km2	Agromatics	Fundamental magnitude in economics
Human work	w	Human labor	Labor relations	Time Recorders	Hour-free, hour-month, etc.	Labor economics	Fundamental magnitude in economics
Biomass	B	Food & Beverage	Man-environ-ment relation-ship	Ecological balance sheets	Ton CO2 / Km2-Day	Ecology	Fundamental in ecology
Energonomy	Ξ	Socio-energetic	Socio-economic effects of energy	Energy balance	Quad	Economic energy	Energy on a social scale
Helio-radiation	Λ	Heliometry	Environmental insulation	Pyro-heliographs	Photo-volts / day	Heliophotometry	Fundamental source of light and energy

Notes on the above table

1. Each magnitude derived from the above (whether physical, social, human, etc.) designated by ∇ (Phoenician nabla) has the dimension

$$\nabla = N^{\upsilon} L^{l} M^{\mu} \ldots \Lambda^{\lambda} \quad \text{(23 factors)}$$

Where the exponents are rational numbers, not all nulls.

2. Some classes of magnitude with which the world of man can be described in quantitative terms are primary or fundamental and others are secondary or derived. They constitute a vector space V on the body Q of the real numbers, and their dimension is 2^{23}. Their "natural" basis is the collection of primary (or unimagnitudinal) sciences that are presented in the previous table.

3. A set of magnitudes, whether fundamental or derived, $X_1, X_2, \ldots, X_h$ generates a discipline D if and only if:

a. Each quantifiable magnitude that is constructed in D, by experimentation or by theoretical reasoning can be expressed dimensionally as $[M] = \prod^{h}_{i=1} X_i^{ai}$, where X_i are fundamental magnitudes and α_i are i=1 nonzero rational numbers;

b. Each magnitude X_i is potentially independent of the others, i.e. $X_1^{d1}\ X_2^{d2} \ldots\ X_{h-1}^{a_{h-1}}\ X_h^{dh}$ = 1 if and only if all the exponents are null.

4. "Counting" is a primary operation of adult man. This was masterfully expressed by the German mathematician Leopold Kronecker in his famous saying: "*Die ganzen Zahlen hat der liebe Gott gemacht, alles andere ist Menschenwerk*." (God made the integers, all else is the work of man.)

5. Let P be an area or part of human knowledge based on measurable facts, phenomena and processes, which is socially and methodologically recognized as a science G, and which refers to objects $X_1,\ldots,X_h$ that generate magnitude M which is the same.

If in the science or discipline G a magnitude M_o is included and expressed as

$$[M_o] = [M_1]^{r_1}\ [M_2]^{r_2}\ \ldots [M_m]^{r_m}$$

where the exponents r_n are non-zero natural numbers, and the collection of powers is said to $UM_n^{r_n}$ form the dimensions M_o of regarding the basis $\{M_1, M_2, \ldots, M_m\}$.

6. A magnitude (or a variable) C is zero-dimensional in a science or discipline G if and only if $[C] = M_1^0\ \ M_2^0\ \ M_m^0$, where M_i are mutually independent fundamental magnitudes of science G.

7. Each discipline or quantifiable science D is a vector space $(X_i,*,\Lambda)$ because among all its elements there is a commutative product $(X_i * X_j \epsilon D)$, and a law of external composition with rational numbers Q (elevation to rational power: $X^r \epsilon D$; $r \epsilon Q$) that satisfy the known algebraic properties of a vector space. As a vector space, D has a base $(Y_1, Y_2, \ldots, Y_m)$ and its neutral element is $Y_1^0\ \ Y_2^0 \ldots Y_m^0$. The latter is the basis of a science without content, which can be called agnosis.

8. Each magnitude of physical, natural, social and human reality, once measured, can be expressed as

$$n \cdot M_1^{r_1}\quad M_2^{r_2} \ldots M_m^{r_m}$$

that is, as the external product of a rational number n with the algebraic product of rational powers of m fundamental magnitudes $M_1 \ldots M_m$. The powers $r_1 = 0 = r_2 = 0 = \cdots = 0 = r_m$ refer to the science of rational numbers (positive and negative), meaning the discipline we call arithmetic.

Unit or fundamental magnitudes

Within the very limited knowledge of this author (G.P.R.), acquired over several decades of study, the following unit magnitudesknown today in the sciences can be identified:

1. Cardinality: the study of natural numbers as a measure of multiplicity or scarcity. This science gives rise to descriptive statistics.
2. Length: the study of distances in a straight line, called longitometry.
3. Ponderal mass (as in the amount of matter in the manner of Lavoisier), which would be called Newtonics.
4. Force:an autonomous magnitude which does not always produce accelerations and whose unitary science should be called dynamonomy.
5. Flatangular amplitude, whose study would be called goniometry.
6. Sharp solid angle, which is rarely studied in physics and whose unitary science (when well developed) would be called the stereometry.
7. Chronometricduration (or the time of physics): measured by chronometers and clocks, studied by Newton and the great physicists, and whose unitary science will be called chronometry. Norbert Wiener calls it Newtonian time.
8. Temperature: like magnitude (which it is), many physicists and physics books ignore it (mistakenly). Its discipline should be called thermometry.
9. Amount of heat: many physics books wrongly identify this with mechanical work, which is not valid, since not every amount of heat can become mechanical work, nor can all mechanical work be converted into heat. Its unitary science should be called thermology.
10. Entropy is a measurement of the disorganization of a system formed by large quantities of discrete elements. The equation $\delta S = \delta Q / T$ is valid only in a reversible process (whose actual duration is infinite) and would only refer to the non-recoverable energy in a process. Its fundamental science is called disorganization.

11.Electric charge: identified by Giovanni Giorgi and whose unit is the coulomb. Its science should be called electrognosia.

12.Magneticity: property independent of the electric charge as evidenced by the existence of permanent magnets and magnetization by contact or friction between a magnet and a piece of neutral steel.

13.Luminosity: an independent property and a characteristic of any visible light source. Its science is called photomatics.

14.Chemical quantity: unit of measurement of the quantity of a chemically reactive substance. Its unitary science is stoichiometry.

15.Human population: fundamental magnitude of the social sciences whose unitary science is called demodynamics.

16.Ordinary money or cash: fundamental magnitude of the economic sciences. Its unitary science is moneterics.

17.Social time (which is independent and not expressible in Newtonian time) is a fundamental magnitude in social and human sciences and economics. It should be called sociochronology.

18.Capital (economic): not to be confused with the magnitude "money," its unitary science is chrematistics.

19.Human work, which is a fundamental magnitude in social science. It is measured in units of civil time multiplied by number of people. Its science is called laboreconomics.

20. Energonomy (productive): fundamental magnitude in economics. Its unit is the quad (quadrillion $Btu = 10^{12} Btu$). Its fundamental science is economic energy, or in Spanish *energoeconomía*.

21.Biomass: fundamental science in ecology. It is measured in diurnal BOD units.

22.Productive land: measured in hectares of land in a territory of recognized and measured productivity, such as Valle del Cauca in Colombia. The unitary science that corresponds to it is agromatics.

23.Insolation or helio-radiation, which is measured in solar radiation for every 24 hours in the tropics. Its fundamental science is the heliophotometry.

Table 1, which appears above, presents these 23 unitary disciplines, which are based on physical, natural, social and human sciences, and are quantifiable.

Conjecture: The developments of biology, molecular physics, earth sciences and other areas, will produce new sciences and technologies. There will be 2^n quantifiable disciplines, being $n > 23$. For example, bioelectricity (such as acupuncture), psychobiology (such as hypnosis), Bergsonian (or vital, coined by Norbert Wiener) time, human work, and other types of knowledge that are just barely emerging today will be rationalizable, quantifiable and manageable in a human, gnoseologically correct and useful way.

With the present state (in 2015 A.D.) of scientific knowledge it can be stated that the 23 disciplines mentioned in the previous table are:

a. Those which can each be constructed from a single fundamental magnitude of reality (physical, human, social, natural), well defined and independent of others, such as physical strength, financial capital, entropy, etc. Therefore, they are called unitary disciplines. The progress of scientific knowledge allows us to expect that its number in the future will be n, greater than the 23 of today.

b. Those whose complete knowledge exhausts the current human scientific knowledge that is measurable and quantifiable today in 2015.

Fundamental dimensions of a discipline

There are disciplines whose measurement operations give results that constitute values belonging to a single magnitude. For example: weighing the packages in a cargo only gives rise to specific values of the ponderal mass; issuing checks to pay for purchases gives rise to the economic size called regular currency; measuring land areas in a territory gives rise to the cadastral area magnitude. Each of these disciplines works with a single magnitude and are therefore called unimagnitudinal magnitudes. Examples are goniometry, statistics, ergonomics, electrognosia, ophelimity, etc.

However, producing the cartography of a territory requires measuring and studying lengths (distances) and angles, so this discipline can be categorized (with many others) among the bimagnitudinal disciplines.

The study of industrial steam engines requires taking into account the ponderal mass variables (steam and fuel), amount of heat, energy (from the machine), duration (time the machine works), money (cost of fuel), and human labor (to attend the machine). This discipline would be called thermomechanics and implies the study of the six magnitudes mentioned, so it is included among the hexamagnitudinal disciplines.

Metrizable magnitudes

El mundo de los seres humanos está constituido por muchas magnitudes, de distinta naturaleza, y que son medibles o contables. Muchas de ellas están en la Naturaleza del Mundo físico en que viven los hombres. Pero otras varias pertenecen a la realidad de las sociedades humanas, como son las disciplinas de la Sociología, la Economía, la Demografía y otras ciencias sociales.

Derived magnitudes or disciplines

By bringing together two unit-baseddisciplines, a binary discipline is constructed. For example, joining the disciplines of human labor and agricultural land into one, a discipline is created which can be called laboragronomy. Combining the discipline of demography with that of energy (what remains to be done) would give us the discipline called social energetics. When there are n unitary disciplines one could construct $\binom{n}{2} = n!/\ [2!\ (n-2)!]$ unitary disciplines as mentioned above.

And one could construct $\binom{n}{3}$ ternary disciplines, and $\binom{n}{4}$ quaternary disciplines, etc. In total, the number of non-empty disciplines that can be constructed is:

$$\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+\binom{n}{3}+\cdots+\binom{n}{n-1}+\binom{n}{n}=\sum_{k=0}^{n}\binom{n}{k}$$

Where $\binom{n}{1} = n$ are the unitary disciplines, etc., up to $\binom{n}{n} = 1$ (single) discipline, which will be universal science. There would be (in theory), $\binom{n}{0} = 1$ (one) empty discipline, which (in theory) would not be constructed with any magnitude of the world of reality and would be called "knowledge without dimensions."

Consequently, the number of non-empty disciplines that could construct all human knowledge would be

$$\sum_{k=0}^{k=n}\binom{n}{k} = \sum_{k=0}^{n}\binom{n}{k}1^{k}1^{n-k} = 2^{n}$$

Today (2015), with the 23 unit disciplines listed above, it is possible to construct

$$\sum_{k=0}^{23}\binom{n}{k} = 2^{23} = 8'388.608$$

disciplines of all dimensions, from arithmetic (as a null-dimensional discipline) to universal science (dimension 23).

The vast majority of these possible sciences have not begun to formalize. In any case, the set of the 2^n possible disciplines constitute, together, the possible scope of the quantifiable knowledge of man.

Pi Theorem of Buckingham (1914), Vaschy (1890) and Riabouchinsky(1911)

This theorem states that in the physical, biological, and socioeconomic sciences—which are quantifiable—the functional relation that links each magnitude X_i with other magnitudes ($X_i = f(X_1, X_2, \dots, X_n)$, necessarily has the form $\emptyset(C_1, C_2, \dots, C_p) = 0$, where C_k are are zero-dimensional monomials of the form $C_k = AX_{1k}^{\alpha ik} \dots X_{nk}^{\alpha nk}$ whose dimension on the basis of the discipline is

$$[C_k] = M_1^0 \dots M_n^0$$

where $M_1, \dots, M_n$ are fundamental magnitudes of the discipline or science in question.

Therefore, according to the implicit function theorem, any of the C_i can be explicitly expressed as

$$C_i = \psi(C_1, C_2, \dots, C_n)$$

But C_1, for example, takes the form

$$C_1 = \alpha_1 X_1^{u_1} \dots X_n^{u_n} = \psi*(C_2, C_3, \dots, C_n)$$

from where:

$$X_1 = k_1 (X_2^{u_2} \cdot X_3^{u_3} \dots . X_n^{u_n})^{-1/u_1} \cdot \psi * (C_2, \dots, C_n)$$

where $k_1 \epsilon \mathbb{R}$, $u_i \epsilon Q$.

This theorem is applied to deduce quantitative laws in the aforementioned sciences using the following algorithm from Lord Rayleigh, which is illustrated with the following problem.

Problem. A phenomenon F (or process, structure or system) has been studied scientifically and comprehensively through experiments, observation, reasoning and common sense, and is framed within the discipline D (hydromechanics, economics, sociometry, etc.). The fundamental magnitudes of the discipline D are $M_1, M_2, \dots M_m$.

1. Previous empirical and critical studies have shown that n variables $x_1, x_2, \dots, x_n$ (and not others) have an incidence in F, and the functional relationship that binds them is sought:

$$\emptyset(x_1, x_2, \dots, x_n) = 0$$

2. The fundamental magnitudes of the science in question are $M_1, M_2, \dots, M_m$

3. The dimensions of the variables x_i in this base are:

$$[x_1] = M_1^{c11}\ M_2^{c12}\ M_3^{c13} \dots M_m^{c1m}$$
$$[x_2] = M_1^{c21}\ M_2^{c22}\ M_3^{c23} \dots M_m^{c2m}$$
$$[x_n] = M_1^{cn1}\ M_2^{c_n2}\ M_3^{cn3} \dots M_m^{cnm}$$

4. According to the Pi Theorem of Buckingham, Vaschy and Riabouchinski, the quantitative law that links the variables $x_1, x_2, \dots, x_n$ presents them as monomial factors that are zero-dimensional of the type

$$Cd = x_1^{P1}\ x_2^{P2} \dots x_n^{Pn}$$

and such that

$$[Cd] = M_1^0\ M_2^0 \dots M_m^0$$

Thus, the n equations with m unknown quantities are

$$p_1 \cdot c_{11} + p_2 \cdot c_{12} + \cdots p_n \cdot c_{1m} = 0$$
$$p_1 \cdot c_{21} + p_2 \cdot c_{22} + \cdots p_n \cdot c_{2m} = 0$$
$$\dots\dots\dots\dots\dots..$$
$$p_1 \cdot c_{n1} + p_2 \cdot c_{n2} + \cdots p_n \cdot c_{nm} = 0$$

Where the numbers c_{ij} are known rational numbers, and p_j are unknowns.

5. In each specific problem, there may be several situations:

a. The simplest case is when $n = m$ and the characteristic of the system is also n. This is a homogeneous linear system that is solved as indicated below.

b. If n is $n = m$ and the characteristic of the system of equations is greater than n and m, every sub-determinant of the coefficients c_{ij} is zero. The system is not solvable. This means that the chosen variables are not compatible with the fundamental quantities that are attributed to them, that is, the equations proposed in number 3 are not valid.

3. If the characteristic of the system of equations is less than or equal to the smaller of the numbers n and m, it means that there is a sub-determinant of the system's coefficients that is nonzero. The system is solvable.

4. If the characteristic r of the system is $r = m < n$ (there are more equations than unknowns), it may or may not have solutions.

5. If the characteristic r of the system is $r = n \leq m$ (there are more unknowns than equations, or there are as many of one as the others), the system is solvable; simply choose one of the unknowns

(for example: p^*) and divide all the equations by p^*. A non-homogeneous system of n equations with $n-1$ unknowns of the form $p_i/p* = \beta_i$, is solved by the method known in linear algebra.

The zero-dimensional polynomial Cd that is sought is

$$Cd = (x_1^{\beta 1}\ x_2^{\beta 2} \ldots x_n^{\beta n})\ p^*$$

meaning the solution sought is

$$\emptyset\ (x_1^{\beta 1}\ x_2^{\beta 2} \ldots x_n^{\beta n}) = 0$$

If one wants to obtain one of the variables, explicitly, let us say x_1, it is cleared as follows: from the previous equation, we get:

$$x_1^{\beta 1}\ x_2^{\beta 2} \ldots x_n^{\beta n} = \varepsilon$$

where ε is a constant that is unknown but not arbitrary. The x_1 is obtained with

$$x_1 = x_2^{-\beta_2/\beta_1}\ x_3^{-\beta_3/\beta_1} \ldots x_n^{-\beta_n/\beta_1} \cdot \varepsilon^{1/\beta_1}$$
$$= \eta\ (x_2^{p_2}\ x_3^{p_3} \ldots x^{p_n})^{1/p_1}$$

Note. In forming the nonhomogeneous system of n unknowns, it may be that some of the exponents of the variables x, when solving the system, are expressed as a linear combination of two of them, i.e. $p_k = a_k p^* + b_k p^{**}$.

In such a case, it will be found that the zero-dimensional monomial sought is

$$Cd = x_1^{\beta_1(a_1 p^* + b_1 p^{**})}\ x_2^{\beta_2(a_2 p^* + b_2 p^{**})} \ldots\ x_n^{\beta_n(a_n p^* + b_n p^{**})}$$

or rather

$$Cd = (x_1^{\beta_1 a_1}\ x_2^{\beta_2 a_2} \ldots x_n^{\beta_n a_n})^{p^*} \cdot (x_1^{\beta_1 b_1}\ x_2^{\beta_2 b_2} \ldots x_n^{\beta_n b_n})^{p^{**}}$$

which means that the products Π sought are products of arbitrary powers of the two zero-dimensional parentheses written above:

$$\Pi_1 = x_1^{\beta_1 a_1} \ldots x_n^{\beta_n a_n}\ ;\ \Pi_2 = x_1^{\beta_1 b_1} \ldots x_n^{\beta_n b_n}$$

Then the relationship that is sought has the form

$$\phi(\Pi_1, \Pi_2) = 0$$

$\Pi_1 = F(\Pi_2) \cdot \alpha$, where α is a dimensionless constant. Then:

$$x_1^{\beta_1 a_1} \ldots x_n^{\beta_n a_n} = \alpha \cdot F\ (x_1^{\beta_1 b_1} \ldots x_n^{\beta_n b_n})$$

And if you want to get explicitly x_1, it follows

$$x_1 = x_2^{-a_2 \beta_2 / a_2 \beta_1} \ldots x_n^{-a_n \beta_n / a_1 \beta_1}$$

$$\cdot \left\{F\left(x_1^{\beta_1 b_1} \ldots x^{\beta_n b_n}\right)\right\}^{1/\beta_1 a_1} \cdot A$$

where A is a real or fractional number, unknown but not arbitrary, to be sought by experimental, numerical or logical methods.

In the above equations, the exponents $u(i, j)$ are known rational numbers, and the unknowns are: $p_1, p_2, \ldots, p_n$.

Depending on the nature of the problem and the empirical knowledge about it, in the above equations there may be fewer variables x_i than basic dimensions, in which case there are fewer unknowns than equations and the system of linear algebraic equations is an over-determined system. In this case the analysis must continue to establish that, although there is an excessive number of equations, all are mutually compatible.

If there are as many unknowns (n) as equations, and since the system of equations is homogeneous, the determinant of the coefficients of the unknowns must be zero, as algebra indicates. In this case, we proceed as follows:

1. Case of n equations and n unknowns p_i:

a. One of the equations is neglected, leaving $n-1$ equations (each equalized to zero and each with n unknowns).

b. We divide each of the remaining equations by one of the unknowns (e.g.: p_1), leaving as new unknowns the ratios p_2/p_1 , p_3/p_1 , … , p_n/p_1, and leaving the column occupied by as a column of constants.

c. This column of constants is moved to the right side of the system, so there is now a non-homogeneous system whose unknowns are now of the form p_i/p_1.

d. Verify that this system has a characteristic equal to $n-1$.

e. Solve this system to calculate the $n-1$ values p_i/p_1.

f. Express a p_2, … , p_n as multiples of p_1 and replace them in each variable p_i/p_1; form the zero-dimensional monomials that can already be built.

2. Case of n equations and m unknowns (where $m > n$):

a. Choose n unknowns p_i (e.g.: $p_1, p_2, \dots . p_n$) as unknowns as variable parameters.

b. Create the system of n equations with n unknowns (p_i) leaving the variable parameters to the right.

c. Solve the system to calculate the n unknowns p_i in function (linear combination) of the $m - n$ variable parameters.

d. Form the zero-dimensional monomials that were sought.

3. Case of n equations and m unknowns (where $n > m$. case very infrequent in the practice of this discipline).

a. Dispense with $m - n$ equations, thereby leaving a homogeneous system of order $n \times n$.

b. Verify that the characteristic of this system is n (independence and compatibility of the system $n \times n$).

c. Substituting each surplus equation into one (or several) of the principals, verify their independence and compatibility with all others.

4. Substitute each exponent p_i already numerically found, or as a linear combination of variable parameters; in the exponents of each variable, intervene in the problem; gather as zero-dimensional monomials groups of variables that have been elevated to the same power. Each of these groups is a zero-dimensional monomial. Of these there are $m - n - 1$. Let be $C_1, C_2, \dots, C_{m-n-1}$ said products.

5. Form the product

$$\prod C_1^{e_1} \cdot \prod C_2^{e_2} \dots \prod C_{m-n-1}^{e_{m-n-1}} .$$

And write the implicit function that binds all the original variables:

$$\psi(C_1, C_2, \dots, C_{m-n-1}) = 0$$

Where each zero-dimensional monomial has the form

$$C = x_1^{\mu_1} \cdot x_2^{\mu_2} \cdot x_h^{\mu_h}$$

6. Choose the monomial C_j^* that contains the variable (of the problem) to be cleared. Suppose that variable is x_j^*.

7. Using the implicit function theorem, clear C_j^* as an explicit function of the other monomials C_j:

$$C_j^* = \emptyset(C_1, \dots, C_{m-n-1})$$

8. In the above equation, clear x_j^*:

$$x_j^* = a\, x_h\ x_k, \dots, x_p\ \emptyset\ (C_1, \dots, C_{m-n-1})$$

where a is a non-arbitrary, three-dimensional constant. However, one must look for other methods or leave it indicated as such.

Example. The preparatory studies of a new industry project have shown that the company's annual profit (R) will be determined by the following: the amount or cost of the investment (A); the annual market value of the product to be produced in the project (M); the depreciation term of the assets (T); and the cost (or profitability) of money (i). This is a matter of calculating the functional relation that expresses the dependence of R on the other variables.

Solution. The fundamental magnitudes of the problem are:

Money:	Δ
Annual physical quantity of the project's product:	Φ
The course of civilian time:	Ω

And the dimensions of the variables considered are:

$[R] = \Delta\, \Omega^{-1}$ $\quad [A] = [R] = \Delta\, \Omega^{\alpha}\ \Phi^{-\alpha}$ (where cost of the factory $= k \times$ (production capacity)$^{\alpha}$

$[M] = \Phi\, \Omega^{-1}$ $\quad [T] = \Omega$

$[F] = \Delta\, \Omega^{-1}$ $\quad [i] = \Omega^{-1}$

A zero-dimensional product of these variables will be

$\prod R^{\lambda}\ A^{\omega} M^{m} T^{n} F^{f} i^{k}$, and its dimensions are:

$$[\Pi] == (\Delta\Omega^{-1})^{\lambda}\ (\Delta\Omega^{\alpha}\Phi^{-\alpha})^{\omega} (\Phi\Omega^{-1})^{m}\ \Omega^{n}\ (\Delta\Omega^{-1})^{f}\ (\Omega^{-1})^{k} = \Omega^0\Delta^0\Phi^0$$

from where:

For Δ: $\lambda + \omega + f = 0$; For Ω: $-\lambda + a\omega - m + n - f - k = 0$ and for Φ: $\alpha\omega + m = 0$

Equations that form the system

$$\lambda + \omega = -f$$

$$-\lambda + \alpha\omega - m = f + k - n$$

$$\alpha\omega - m = 0$$

and the system of its coefficients is

$$\begin{vmatrix} 1 & 1 & 0 \\ -1 & \alpha & -1 \\ 0 & \alpha & -1 \end{vmatrix} = (1+\alpha)$$

and their solutions are

$$\lambda = \frac{(2\alpha+1)f + k - n}{1+\alpha}, \quad m = \frac{\alpha(k+n)}{1+\alpha}, \quad \omega = \frac{k+m}{1+\alpha}$$

Where the product P_i sought is

$$\prod = R^{-(2\alpha+1)/(1+\alpha)}\, F) \left[(R^{-1}AM^{\alpha})^{1/(1+\alpha)}\ \ i\right]^k \left[(RAM)^{1/(1+\alpha)}T\right]^n$$

which is to say that the product Π sought is the product of arbitrary powers of the three zero-dimensional products that appear in the parentheses.

Given its nature and meaning, the number α is real and positive, so that Π is the zero-dimensional product

$$(R^{-(2\alpha+1)(f+k-n)}\, A^{k+n}\, M^{\alpha k+n})^{1/(1+\alpha)}\, (F^f T^n)^{1/(1+\alpha)}$$

which is a product of the two monomials (zero-dimensional)

$$Cd_1 = R^{-(2\alpha+1)(f+k-n)} A^{k+n}\, M^{\alpha k+n}$$

$$Cd_2 = F^f T^n$$

According to the pi theorem, the solution sought has the form

$$\Phi(Cd_1, Cd_2) = 0$$

and according to the implicit function theorem of:

$$Cd_1 = k\, \varphi(Cd_2)$$

where k is an indeterminate but not arbitrary constant, and φ is an undetermined function of Cd_2 but not arbitrary. That is:

$$R^{-(2\alpha+1)(f+k-n)}\, A^{k+n}\, M^{\alpha k+n} = k \cdot \varphi\,(F^f T^n)$$

$$R = c \cdot \left[A^{k+n} M^{\alpha k+n}\right]^{-1/(2\alpha+1)(f+k-n)} \cdot \varphi(F^f T^n)$$

If a utility (R) is required which is increasing or proportional to the amount of the investment, is needed $R/A \geq 1$. And since it is also necessary that the annual profit is increasing or proportional to the size of the market (M), it is also required that R/M is $R/M \geq 1$. Microeconomics teaches that α is always α < 1, thus $\alpha k + n < k + n$.

For microeconomic reasons, annual profit must be proportionally greater than the cost of money and depreciation; then, the product FT should be less than profitability.

REFERENCES

Preceding note. Most of the fundamental ideas, definitions and propositions set forth in this document have been the product of the author's studies, reflections and writings on the theory of knowledge, dimensional analysis and abstract algebra. For this reason, the bibliography that has been consulted to write this document,while not necessarily abundant,manages to be very substantial.

Boorstin, D. (1998). The Seekers. New York. Random House. 298 p.

Chauvineaus, J. (1957). La logique moderne. Paris. Presses Universitaires de France. 126 p.

Cohen, M.; Ernest, N. (1968). Introducción a la Lógica y al método científico. Buenos Aires. Amorrortu Editores. 284 p.

Daval, S.; Bernard, G. (1964). Filosofía de las Ciencias Buenos Aires. Editorial El Ateneo. 496 p.

Ferrater Mora, J. (1979). De la materia a la razón. Madrid. Alianza Editorial. 217 p.

Geymonat, L. (S.F.) Filosofía y Filosofía de la ciencia. Barcelona. Editorial Labor S.A. 170 p.

Geymonat, L. (1985). Historia de la Filosofía y de la Ciencia. 3 vols. Barcelona. Editorial Crítica. Vol. 1, 312 p.

Hazlitt, H. (1969). El pensar como ciencia. Buenos Aires. Editorial Nova. 205 p.

Hessen, H. (1925). Teoría del conocimiento. Colonia y Bogotá. Gráficas Modernas. 157 p.

Langhaar, H.C. (1956). Analyse Dimensionelle et Théorie des Maquettes. (Traducción del inglés). Paris. Editorial Dunod. 230 p.

Popper, K R. (1961). The logic of scientific Discovery. New York. Science Editions, Inc. 477 p.

Poveda Ramos, G. (2008). Modelo matemático y dimensional para el planteamiento óptimo de industrias de procesos. Medellín. Instituto Tecnológico metropolitano. 143 p.

Russell, B. (1959). El conocimiento humano. Madrid. Editorial Tausus. 2 volúmenes. 310 y 354 pp.

Simmel, G. (1961). Problemas fundamentales de la Filosofía. México. Editorial UTEHA. 165 p.

Tarski, A. (1951). Introducción a la Lógica y a la Metodología de las ciencias deductivas. Buenos Aires. Espasa-Calpe Argentina S.A. 237 p.

TO REFERENCE THIS ARTICLE /
PARA CITAR ESTE ARTÍCULO /
PARA CITAR ESTE ARTIGO /

Poveda Ramos, G. (2016). Generalized Dimensional Analysis. *Revista EIA*, 13(25), January-June, pp. 13-27. [Online]. Available at: DOI: http:/dx.doi.org/10.14508/reia.2016.13.25.13-27